Int. J. Solids Structures Vol. 16, pp. 329–335
© Pergamon Press Ltd., 1980. Printed in Great Britain

0020-7683/80/0401-0329/$02.00/0

GREEN'S FUNCTION FOR A SEMICIRCULAR PLATE†

A. K. NAGHDI

Division of Engineering, Purdue School of Engineering and Technology at Indianapolis, IN 46205, U.S.A.

(*Received* 12 *January* 1979; *in revised form* 3 *April* 1979)

Abstract—The exact closed form solution is derived for the displacement of a thin elastic semicircular plate acted upon by a concentrated force at an arbitrary point. It is assumed that the semicircular boundary of the plate is clamped, and that its diametral boundary is simply supported. The derivation is accomplished in the following manner. First, a singularity function is employed in order to obtain a closed form solution for the displacement of the semicircular plate subject to a circumferential sinusoidal line load. Thus a solution for arbitrary circumferential line loading of the plate is derived in the form of a Fourier series. Next, the series solution for the particular case of a concentrated force is arranged into components which can be summed in closed forms. Finally it has been shown that with the exception of the point of application of the concentrated load, the obtained Green's function and its derivatives of any order are continuous in the region. Numerical values of dimensionless stress couples at various points of the region are presented for three different positions of the concentrated force.

INTRODUCTION

The solution for the displacement of a circular plate subjected to a discontinuous load has been derived in various investigations. One of the early investigations is due to Michell[1]. Employing the method of inversion, he derived the closed form solution for the displacement of a clamped circular plate acted upon by a concentrated force. Later on Melan[2] obtained an identical solution with the use of bipolar coordinates. More recently Bassali[3], Dundurs and Lee[4], Yu and Pan[5], Lee[6], Amon and Widera[7] and Williams and Brinson[8] investigated the cases of certain plate problems involving concentrated forces. Most of the latter authors utilized Michell's closed form expression in the analysis of their solutions.

In the present investigation the closed form Green's function for a semicircular plate, clamped around the curved edge and simply-supported along its diameter, is obtained. The technique of solution is somewhat different from those employed by the previous investigators.

METHOD OF SOLUTION

Consider a semicircular plate with radius R simply supported along its diameter and clamped around its curved edge. A set of dimensionless polar coordinates $\rho = r/R$ and θ is chosen such that its origin is at the center of the circle, and that the lines $\theta = 0$, $\theta = \pi$ coincide with the diameter of the plate as shown in Fig. 1. The differential equation for the transverse

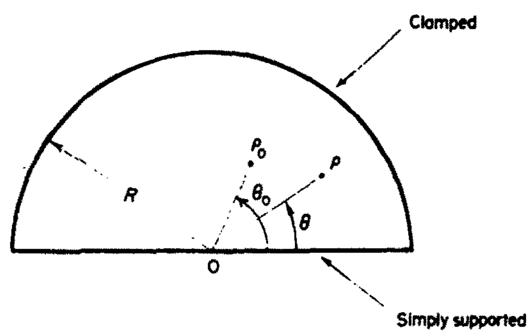

Fig. 1. Semi-circular plate subject to a concentrated force. (ρ_0, θ_0) and (ρ, θ) are respectively the polar coordinates of the positions of the concentrated load and a point in the region.

†The author wishes to thank the Department of Computer Services at IUPUI and the Indiana University Computing Network for providing the computer time on the CDC 6600 that was used on this project.

displacement w in a thin elastic plate is given by

$$\left.\begin{aligned}\nabla^2\nabla^2\bar{w} &= \frac{R^3 p}{D}, \qquad \bar{w} = \frac{w}{R},\\ \nabla^2 &\equiv \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2},\end{aligned}\right\} \tag{1}$$

in which D is the plate modulus, and p is the transverse load per unit area of the plate. For the plate under consideration the edge conditions are:

$$\left.\begin{aligned}\bar{w} &= 0\\ \frac{\partial\bar{w}}{\partial\rho} &= 0\end{aligned}\right\} \qquad \text{at } \rho = 1; \tag{2}$$

$$\left.\begin{aligned}\bar{w} &= 0\\ \nu\frac{\partial^2\bar{w}}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\bar{w}}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\bar{w}}{\partial\theta^2} &= 0\end{aligned}\right\} \qquad \text{at } \theta = 0, \quad \theta = \pi. \tag{3}$$

Here in relations (3) ν is Poisson's ratio. Now assume that the semicircular plate is acted upon by the following sinusoidal line load on the circumferential line $\rho = \rho_0 < 1$:

$$p_n(\rho, \theta) = p_n^*\delta(\rho - \rho_0)\sin n\theta \qquad n = 1, 2, 3, \ldots, \tag{4}$$

in which p_n^* is a constant, and $\delta(\rho - \rho_0)$ is the unit impulse function. Relation (4) is inserted in eqn (1) to yield

$$\nabla^2\nabla^2\bar{w} = \frac{R^3}{D}p_n^*\delta(\rho - \rho_0)\sin n\theta. \tag{5}$$

A solution in the form

$$\bar{w}_n = f_n(\rho)\sin n\theta \tag{6}$$

is now substituted in eqn (5) to give

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} + \frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho} - \frac{n^2}{\rho^2}\right)\left(\frac{\mathrm{d}^2 f_n}{\mathrm{d}\rho^2} + \frac{1}{\rho}\frac{\mathrm{d}f_n}{\mathrm{d}\rho} - \frac{n^2}{\rho^2}f_n\right) = \frac{R^3}{D}p_n^*\delta(\rho - \rho_0). \tag{7}$$

The complementary solution f_{nc} of the differential equation (7) is derived in the usual way, and its particular integral f_{np} is obtained with the known method of variation of parameters [9]. In the derivation of f_{np} it is considered that the function must have continuous derivatives up to the second order at $\rho = \rho_0$. These complementary and particular solutions are

$$f_{nc} = A_n\rho^{n+2} + C_n\rho^n \qquad n = 1, 2, 3, \ldots, \tag{8}$$

in which A_n and C_n are the unknown constants of integration, and

$$\left.\begin{aligned}f_{1p} &= \frac{p_1^*\rho R^3}{4D}\left[\frac{1}{2}(\rho^2 - \rho_0^2) - \rho_0^2\ln\frac{\rho}{\rho_0}\right] - \frac{p_1^*R^3}{4\rho D}\left[\frac{1}{4}(\rho^4 - \rho_0^4) - \frac{1}{2}\rho_0^2(\rho^2 - \rho_0^2)\right] \quad \rho \geq \rho_0,\\ f_{1p} &= 0 \qquad \rho < \rho_0,\end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned}f_{np} &= \frac{p_n^*R^3}{4nD}\left[\frac{1}{2(n+1)}\rho_0^{-n+1}\rho^{n+2} + \frac{1}{2(n-1)}\rho_0^{n+1}\rho^{-n+2}\right.\\ &\qquad \left. - \frac{1}{2(n-1)}\rho_0^{-n+3}\rho^n - \frac{1}{2(n+1)}\rho_0^{n+3}\rho^{-n}\right] \quad \rho \geq \rho_0,\\ f_{np} &= 0 \qquad \rho \leq \rho_0, \qquad n = 2, 3, 4, \ldots\end{aligned}\right\} \tag{10}$$

The general solution

$$(f_{nc}+f_{np})\sin n\theta$$

which automatically satisfies the boundary conditions (3), is now substituted in the edge conditions (2) in order to give

$$\begin{aligned} A_1 &= -\frac{1}{2}(\alpha_1-\beta_1) \\ C_1 &= -\frac{1}{2}(\beta_1-3\alpha_1), \end{aligned} \tag{11}$$

where

$$\begin{aligned} \alpha_1 &= -\frac{p_1^* R^3}{4D}\left[\frac{1}{2}(1-\rho_0^2)-\rho_0^2\ln\frac{1}{\rho_0}\right]+\frac{p_1^* R^3}{4D}\left[\frac{1}{4}(1-\rho_0^4)-\frac{1}{2}\rho_0^2(1-\rho_0^2)\right], \\ \beta_1 &= -\frac{p_1^* R^3}{4D}\left[\frac{1}{2}(3-\rho_0^2)-\rho_0^2\left(\ln\frac{1}{\rho_0}+1\right)\right]+\frac{p_1^* R^3}{4D}\left[\frac{1}{4}(3+\rho_0^4)-\frac{1}{2}\rho_0^2(1+\rho_0^2)\right], \end{aligned} \tag{12}$$

and

$$A_n = -\frac{1}{2}(n\alpha_n-\beta_n), \qquad c_n = -\frac{1}{2}[\beta_n-(n+2)\alpha_n], \tag{13}$$

in which

$$\begin{aligned} \alpha_n &= -\frac{p_n^* R^3}{4nD}\left[\frac{1}{2(n+1)}\rho_0^{-n+1}+\frac{1}{2(n-1)}\rho_0^{n+1}+\frac{1}{2(n-1)}\rho_0^{n+1}-\frac{1}{2(n-1)}\rho_0^{-n+3}-\frac{1}{2(n+1)}\rho_0^{n+3}\right], \\ \beta_n &= -\frac{p_n^* R^3}{4nD}\left[\frac{n+2}{2(n+1)}\rho_0^{-n+1}+\frac{-n+2}{2(n-1)}\rho_0^{n+1}-\frac{1}{2}\rho_0^{-n+3}-\frac{1}{2(n-1)}\rho_0^{-n+3}+\frac{1}{2}\rho_0^{n+3}\right. \\ &\quad \left.-\frac{1}{2(n+1)}\rho_0^{n+3}\right], \qquad n=2,3,4,\dots \end{aligned} \tag{14}$$

In view of the principle of superposition the aforementioned solutions are combined in order to yield the solution for an arbitrary line loading of the plate at $\rho=\rho_0$. Thus

$$\bar{w}=(f_{1c}+f_{1p})\sin\theta+\sum_{n=2,3,\dots}^{\infty}(f_{nc}+f_{np})\sin n\theta. \tag{15}$$

The constants p_n^* contained in (15) are

$$p_n^*=\frac{2}{\pi R}\int_0^{\pi}\bar{p}(\theta)\sin n\theta\,\mathrm{d}\theta, \tag{16}$$

in which $\bar{p}(\theta)$ is the intensity of the line load per unit length of the circumference at the radius $r_0=R\rho_0$. For the particular case of a uniform segmental line load with intensity $\bar{p}$, whose midpoint is at ρ_0, θ_0 and is confined in angle θ^*, the following result is obtained:

$$\begin{aligned} p_n^* &= \frac{2}{\pi R}\int_{\theta_0-(\theta^*/2)}^{\theta_0+(\theta^*/2)}\bar{p}\sin n\theta\,\mathrm{d}\theta=\frac{4\bar{p}}{n\pi R}\sin n\theta_0\sin n\frac{\theta^*}{2} \\ &\pi>\theta_0>0, \qquad n=1,2,3,\dots \end{aligned} \tag{17}$$

As θ^* tends to zero while the total load $\bar{p}r_0\theta^*$ approaches P the limiting value of p_n^* for the concentrated load case becomes

$$p_n^* = \frac{2P}{\pi R^2 \rho_0} \sin n\theta_0, \qquad n = 1, 2, 3, \ldots. \tag{18}$$

The value of p_n^* in eqn (18) is now inserted in the previously obtained relations. After a few simplifications the Fourier series form of the Green's function is written:

$$\bar{W} = \bar{W}_1 + \sum_{n=2}^{\infty} \bar{W}_n, \tag{19}$$

in which

$$\left.\begin{aligned}\bar{W}_1 = \frac{PR}{4\pi D}\Big[&-\frac{1}{4\rho_0}(\rho_0^2-1)^2\rho^3 - \frac{1}{2}\rho_0(2\ln\rho_0 + 1 - \rho_0^2)\rho \\ &+\frac{1}{2}\frac{\rho}{\rho_0}(\rho^2-\rho_0^2) - \rho\rho_0\ln\frac{\rho}{\rho_0} - \frac{1}{4\rho\rho_0}(\rho^4-\rho_0^4) + \frac{\rho_0}{2\rho}(\rho^2-\rho_0^2)\Big](\cos\phi_2 - \cos\phi_1) \qquad \rho \geq \rho_0,\end{aligned}\right\} \tag{20}$$

$$\bar{W}_1 = \frac{PR}{4\pi D}\left[-\frac{1}{4\rho_0}(\rho_0^2-1)^2\rho^3 - \frac{1}{2}\rho_0(2\ln\rho_0 + 1 - \rho_0^2)\rho\right](\cos\phi_2 - \cos\phi_1) \qquad \rho < \rho_0, \tag{21}$$

$$\phi_1 = \theta + \theta_0, \qquad \phi_2 = \phi - \theta_0, \tag{22}$$

and

$$\left.\begin{aligned}\bar{W}_n = -\frac{PR}{8\pi Dn}\Big[&-\rho_0^n\rho^{n+2} + \frac{n}{n+1}\rho_0^{n+2}\rho^{n+2} - \rho_0^{n+2}\rho^n + \frac{n}{n-1}\rho_0^n\rho^n - \frac{1}{n-1}\rho_0^n\rho^{-n+2} \\ &+\frac{1}{n+1}\rho_0^{n+2}\rho^{-n}\Big](\cos n\phi_2 - \cos n\phi_1) \qquad \rho \geq \rho_0, \quad n = 2, 3, 4, \ldots,\end{aligned}\right\} \tag{23}$$

$$\left.\begin{aligned}\bar{W}_n = -\frac{PR}{8\pi Dn}\Big[&-\rho_0^n\rho^{n+2} + \frac{n}{n+1}\rho_0^{n+2}\rho^{n+2} - \rho_0^{n+2}\rho^n + \frac{n}{n-1}\rho_0^n\rho^n + \frac{1}{n+1}\rho_0^{-n}\rho^{n+2} \\ &-\frac{1}{n-1}\rho_0^{-n+2}\rho^n\Big](\cos n\phi_2 - \cos n\phi_1) \qquad \rho < \rho_0, \quad n = 2, 3, 4, \ldots\end{aligned}\right\} \tag{24}$$

It is noted that

$$\left.\begin{aligned}&[\bar{W}_1(\rho, \theta, \rho_0, \theta_0)]_{(\rho \geq \rho_0)} = [\bar{W}_1(\rho_0, \theta, \rho, \theta_0)]_{(\rho < \rho_0)}, \\ &[\bar{W}_n(\rho, \theta, \rho_0, \theta_0)]_{(\rho \geq \rho_0)} = [\bar{W}_n(\rho_0, \theta, \rho, \theta_0)]_{(\rho < \rho_0)},\end{aligned}\right\} \tag{25}$$

as they should be.

It has been shown previously [10–13] that series similar to those involved in relations (23) and (24) have closed form sums:

$$\left.\begin{aligned}&\sum_{k=1}^{\infty}\frac{e^{-k\xi}}{k}\cos k\phi = F_1(\xi, \phi), \\ &F_1(\xi, \phi) = \frac{1}{2}\ln\frac{\cosh\xi - 1}{\cosh\xi - \cos\phi} - \ln(1 - e^{-\xi}) \\ &\xi > 0, \\ &\sum_{k=1}^{\infty}\frac{e^{-k\xi}}{k}\sin k\phi = F_2(\xi, \phi), \\ &F_2(\xi, \phi) = -\frac{\phi}{2} + \arctan\left[\frac{(1+\cosh\xi)\tan(\phi/2)}{\sinh\xi}\right] \\ &\xi > 0, \qquad \pi > \phi > -\pi, \\ &F_2(\xi, \phi) = -F_2[\xi, (2\pi - \phi)] \\ &\xi > 0, \qquad 2\pi > \phi > \pi.\end{aligned}\right\} \tag{26}$$

Since $\rho_0 < 1$ it is obvious that terms in $\bar{W}_n$ can be written to the forms given by identities (26).

For example for $\rho > \rho_0$ one writes:

$$\left.\begin{aligned}
&\sum_{n=2}^{\infty} \frac{\rho_0^{n+2}\rho^{-n}}{n(n+1)} \cos n\phi_2 = \\
&\rho_0^2 \sum_{n=2}^{\infty} \frac{(\rho_0/\rho)^n}{n} \cos n\phi_2 - \rho_0^2 \sum_{n=2}^{\infty} \frac{(\rho_0/\rho)^n}{n+1} \cos n\phi_2 \\
&= \rho_0^2 \sum_{n=2}^{\infty} \frac{e^{-n\xi_2}}{n} \cos n\phi_2 - (\rho\rho_0) \sum_{m=3}^{\infty} \frac{e^{-m\xi_2}}{m} \cos m\phi_2 \\
&- (\rho\rho_0) \sin\phi_2 \sum_{m=3}^{\infty} \frac{e^{-m\xi_2}}{m} \sin m\phi_2, \qquad \xi_2 = -\ln\frac{\rho_0}{\rho} > 0, \\
&\qquad\qquad\qquad\qquad\qquad\qquad m = n+1.
\end{aligned}\right\} \tag{27}$$

The other series in eqns (23) and (24) can be arranged in the same manner. Thus, in view of identities (26) the Green's function in closed form can be written as:

$$\left.\begin{aligned}
\bar{W} &= \frac{PR}{4\pi D}\left\{-\frac{1}{4\rho_0}(\rho_0^2-1)^2\rho^3 - \frac{1}{2}\rho_0(2\ln\rho_0 + 1 - \rho_0^2)\rho + \frac{1}{4}\frac{\rho^3}{\rho_0} - \rho\rho_0\ln\frac{\rho}{\rho_0}\right\}(\cos\phi_2 - \cos\phi_1) \\
&- \frac{PR}{8\pi D}\Big\{-(\rho^2+\rho_0^2)[F_1(\xi_1,\phi_2) - F_1(\xi_1,\phi_1) - \rho\rho_0(\cos\phi_2 - \cos\phi_1)] + 2\rho\rho_0[\cos\phi_2 F_1(\xi_1,\phi_2) \\
&- \cos\phi_1 F_1(\xi_1,\phi_1) - \cos\phi_2 F_1(\xi_2,\phi_2) + \cos\phi_1 F_1(\xi_2,\phi_1)] \\
&+ (\rho^2+\rho_0^2)[F_1(\xi_2,\phi_2) - F_1(\xi_2,\phi_1)] - \rho\rho_0(\cos\phi_2 - \cos\phi_1) - \frac{1}{2}(\rho\rho_0)^3(\cos\phi_2 - \cos\phi_1)\Big\} \\
&\qquad \xi_1 = -\ln\rho\rho_0, \qquad \rho > \rho_0,
\end{aligned}\right\} \tag{28}$$

$$\left.\begin{aligned}
\bar{W} &= \frac{PR}{4\pi D}\left\{-\frac{1}{4\rho_0}(\rho_0^2-1)^2\rho^3 - \frac{1}{2}\rho_0(2\ln\rho_0 + 1 - \rho_0^2)\rho + \frac{1}{4}\frac{\rho^3}{\rho_0}\right\}(\cos\phi_2 - \cos\phi_1) \\
&- \frac{PR}{8\pi D}\Big\{-(\rho^2+\rho_0^2)[F_1(\xi_1,\phi_2) - F_1(\xi_1,\phi_1) - \rho\rho_0(\cos\phi_2 - \cos\phi_1)] + 2\rho\rho_0[\cos\phi_2 F_1(\xi_1,\phi_2) \\
&- \cos\phi_1 F_1(\xi_1,\phi_1) - \cos\phi_2 F_1(\xi_3,\phi_2) + \cos\phi_1 F_1(\xi_3,\phi_1)] \\
&+ (\rho^2+\rho_0^2)[F_1(\xi_3,\phi_2) - F_1(\xi_3,\phi_1)] - \rho\rho_0(\cos\phi_2 - \cos\phi_1) - \frac{1}{2}(\rho\rho_0)^3(\cos\phi_2 - \cos\phi_1)\Big\} \\
&\qquad \xi_3 = -\ln\frac{\rho}{\rho_0}, \qquad \rho < \rho_0
\end{aligned}\right\} \tag{29}$$

It shall be shown now that in both regions $\rho > \rho_0$ and $\rho < \rho_0$ $\bar{W}$ has the same functional form. Thus, excluding the point of application of the concentrated load the value of $\bar{W}$ and its derivatives of any order may be obtained in the region. It is easily seen from identities (26) that

$$\begin{aligned}
F_1(\xi_2,\phi_2) - F_1(\xi_2,\phi_1) &= F_1(\xi_3,\phi_2) - F_1(\xi_3,\phi_1) \\
&= \ln\left[\frac{\rho^2+\rho_0^2-2\rho\rho_0\cos\phi_1}{\rho^2+\rho_0^2-2\rho\rho_0\cos\phi_2}\right]^{1/2}, \qquad \text{Excluding}\begin{cases}\rho = \rho_0, \\ \phi_2 = 0.\end{cases}
\end{aligned} \tag{30}$$

Considering eqn (30), it is noted that all terms in relations (28) and (29) with the exception of

$$-2\rho\rho_0\ln\frac{\rho}{\rho_0} - 2\rho\rho_0[-\cos\phi_2 F_1(\xi_2,\phi_2) + \cos\phi_1 F_1(\xi_2,\phi_1)] \tag{31}$$

in (28) and

$$-2\rho\rho_0[-\cos\phi_2 F_1(\xi_3,\phi_2) + \cos\phi_1 F_1(\xi_3,\phi_1)] \tag{32}$$

in (29) are identical. Utilizing relations (26) and (30), it is not too difficult to show that the expressions (31) and (32) are the same. Thus, the Green's function given in relations (28) and (29) may be written in the following single expression:

$$\left.\begin{aligned}\bar{W} = -\frac{PR}{8\pi D}\Bigg\{&2\rho\rho_0 \ln\rho(\cos\phi_2 - \cos\phi_1) - (\rho^2+\rho_0^2)\ln\left[\frac{1+(\rho\rho_0)^2 - 2\rho\rho_0\cos\phi_1}{1+(\rho\rho_0)^2-2\rho\rho_0\cos\phi_2}\right]^{1/2} \\ &+2\rho\rho_0\Bigg[-\cos\phi_2\ln(1+\rho^2\rho_0^2-2\rho\rho_0\cos\phi_2)^{1/2}+\cos\phi_1\ln(1+\rho^2\rho_0^2-2\rho\rho_0\cos\phi_1)^{1/2} \\ &+\cos\phi_2\ln\left(1+\frac{\rho_0^2}{\rho^2}-2\frac{\rho_0}{\rho}\cos\phi_2\right)^{1/2} - \cos\phi_1\ln\left(1+\frac{\rho_0^2}{\rho^2}-2\frac{\rho_0}{\rho}\cos\phi_1\right)^{1/2}\Bigg] \\ &+(\rho^2+\rho_0^2)\ln\left[\frac{1+(\rho_0^2/\rho^2)-2(\rho_0/\rho)\cos\phi_1}{1+(\rho_0^2/\rho^2)-2(\rho_0/\rho)\cos\phi_2}\right]^{1/2}\Bigg\}, \\ &\text{Excluding}\begin{cases}\rho=\rho_0 \\ \phi_2 = 0\end{cases}.\end{aligned}\right\} \quad (33)$$

NUMERICAL RESULTS

Numerical values of dimensionless stress couples:

$$\bar{M}_\rho = \frac{\partial^2\bar{w}}{\partial\rho^2} + \frac{\nu}{\rho}\frac{\partial\bar{w}}{\partial\rho} + \frac{\nu}{\rho^2}\frac{\partial^2\bar{w}}{\partial\theta^2}, \quad (34)$$

$$\bar{M}_\theta = \nu\frac{\partial^2\bar{w}}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\bar{w}}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\bar{w}}{\partial\theta^2}, \quad (35)$$

are obtained at several points of the region for three different positions of the concentrated force P. In Tables 1 and 2 the values of $\bar{M}_\rho$ and $\bar{M}_\theta$ at various points of the clamped edge $\rho = 1$ are presented for the cases of $\rho_0 = 0.5$, $\theta_0 = 0.5$ rad., and $\rho_0 = 0.4$, $\theta_0 = 1$ rad. respectively. In Figs. 2 and 3 the values of $\bar{M}_\rho$ and $\bar{M}_\theta$ are plotted vs θ for different radii $\rho = Rr$ for the case of $\rho_0 = 0.3$, $\theta_0 = 0.8$ rad.

CONCLUSION

The technique of singularity functions employed in this investigation can be utilized in order to obtain a closed form solution for Poisson's equation involving a source function in a semicircular region subject to a homogeneous boundary condition. The method may also be applied to the cases of a sector of a plate, and a circular cylindrical panel subject to various edge conditions. However, the derivation of closed form Green's functions for these cases, if possible, may need more investigation.

Table 1. Non-dimensional values of stress couples $\bar{M}_\rho$ and $\bar{M}_\theta$ at $\rho = 1$ vs $\theta = k(\pi/12)$ for the case of $\rho_0 = 0.5$, $\theta_0 = 0.5$ rad. and $\nu = 0.3$.

k	$\bar{M}_\rho$	$\bar{M}_\theta$
0	0	0
1	0.075845	0.022754
2	0.117507	0.035252
3	0.107893	0.032368
4	0.076533	0.022960
5	0.049841	0.014952
6	0.032207	0.009662
7	0.021072	0.006321
8	0.013871	0.004161
9	0.008968	0.002691
10	0.005388	0.001617
11	0.002533	0.000760
12	0	0

Table 2. Non-dimensional values of stress couples $\bar{M}_\rho$ and $\bar{M}_\theta$ at $\rho = 1$ vs $\theta = k(\pi/12)$ for the case of $\rho_0 = 0.4$, $\theta_0 = 1$ rad. and $\nu = 0.3$

k	$\bar{M}_\rho$	$\bar{M}_\theta$
0	0	0
1	0.037560	0.011268
2	0.075000	0.022500
3	0.106201	0.031860
4	0.118812	0.035644
5	0.108058	0.032418
6	0.084710	0.025413
7	0.061114	0.018334
8	0.042134	0.012040
9	0.027819	0.008346
10	0.016868	0.005060
11	0.007960	0.002388
12	0	0

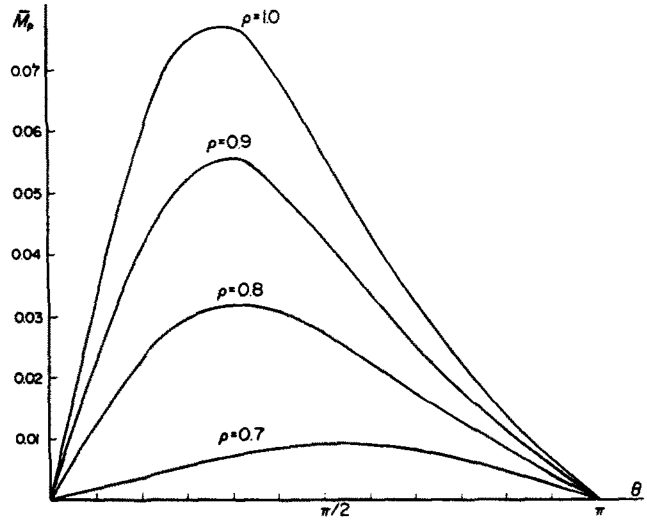

Fig. 2. Variation of $\bar{M}_\rho$ vs θ for various radii, for the case of $\rho_0 = 0.3$, $\theta_0 = 0.8$, $\nu = 0.3$.

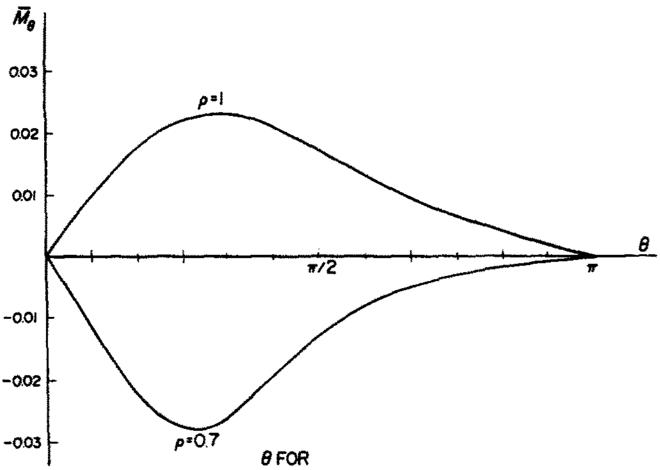

Fig. 3. Variation of $\bar{M}_\theta$ vs two different radii for the case of $\rho_0 = 0.3$, $\theta_0 = 0.8$, $\nu = 0.3$.

REFERENCES

1. J. H. Michell, The flexure of a circular plate. *Proc. London Math. Soc.* **34**, 223–228 (1902).
2. E. Melan, Die durchbiegung einer exzentrish durch eine cinzellast belasteten kreisplatte. *Der Eisenbau* **11**, 190–192 (1920).
3. W. A. Bassali, The transverse flexure of thin elastic plates supported at several points. *Proc. Cambridge Phil. Soc.* **53**, 728–743 (1957).
4. J. Dundurs and T. M. Lee, Flexure by a concentrated force of the infinite plate on a circular support. *J. Appl. Mech.* **2**, 225–231 (1963).
5. J. C. L. Yu and H. H. Pan, Uniformly loaded circular plate supported at discrete points. *Int. J. Mech. Sci.* **8**, 333–340 (1966).
6. T. M. Lee, Fixture of circular plate by concentrated force. *Bibliog. Diags. Am. Soc. CE Proc.* **94** [EM 3 No. 6006], 841–855 (1968).
7. R. Amon and O. E. Widera, Clamped plate under a concentrated force. *Diag. AIAA J.* **7**, 151–153 (1969).
8. R. Williams and H. F. Brinson, Circular plate on multipoint supports. *Bibliog. Diags. Franklin Institute J.* **297**, 429–447 (1974).
9. F. H. Miller, *Partial Differential Equations*, 11th Edn. Wiley, New York (1965).
10. T. J. I. A. Bromwich, *An Introduction to the Theory of Infinite Series*. Macmillan, New York (1965).
11. L. V. Kantorovich and V. I. Krylov, *Approximate Methods of Higher Analysis*. Interscience (1964).
12. A. K. Naghdi, On the convergence of series solutions for a short beam. *J. Appl. Mech.* **41**, 530–531 (1974).
13. A. K. Naghdi and J. M. Gersting, Jr., The effect of a transverse shear acting on the edge of a circular cutout in a simply supported circular cylindrical shell. *Ingenieur-Archiv.* **42**, 141–150 (1973).